Some Generic Properties of Level Spacing Distributions of 2D Real Random Matrices

Siegfried Grossmann[a] and Marko Robnik[b]

[a] Fachbereich Physik der Philipps-Universität, Renthof 6, D-35032 Marburg, Germany
[b] CAMTP – Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, SI-2000 Maribor, Slovenia

Reprint requests to Prof. S. G. or Prof. M. R.; E-mail: Grossmann@physik.uni-marburg.de or Robnik@uni-mb.si

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We study the level spacing distribution $P(S)$ of 2D real random matrices both symmetric as well as general, non-symmetric. In the general case we restrict ourselves to Gaussian distributed matrix elements, but different widths of the various matrix elements are admitted. The following results are obtained: An explicit exact formula for $P(S)$ is derived and its behaviour close to $S = 0$ is studied analytically, showing that there is linear level repulsion, unless there are additional constraints for the probability distribution of the matrix elements. The constraint of having only positive or only negative but otherwise arbitrary non-diagonal elements leads to quadratic level repulsion with logarithmic corrections. These findings detail and extend our previous results already published in a preceding paper. For *symmetric* real 2D matrices also other, non-Gaussian statistical distributions are considered. In this case we show for arbitrary statistical distribution of the diagonal and non-diagonal elements that the level repulsion exponent ρ is always $\rho = 1$, provided the distribution function of the matrix elements is regular at zero value. If the distribution function of the matrix elements is a singular (but still integrable) power law near zero value of S, the level spacing distribution $P(S)$ is a fractional exponent power law at small S. The tail of $P(S)$ depends on further details of the matrix element statistics. We explicitly work out four cases: the uniform (box) distribution, the Cauchy-Lorentz distribution, the exponential distribution and, as an example for a singular distribution, the power law distribution for $P(S)$ near zero value times an exponential tail.

Key words: Random Matrix Theory; Level Spacing Distribution; Non-Normal Matrices; Level Repulsion; Non-Gaussian Level Statistics.

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